

Assignment – II
MATHEMATICS – III
SEMESTER-IV (CS/IT), Paper Code: M401

(Multiple Choice Type Questions)

- 1) i) If A and B are two events with $P(A) = 0.4$, $P(B) = 0.3$ and $P(A \cap B) = 0.2$, then $P(A^c \cap B)$ is
 (a) 0.1 (b) 0.2 (c) 0.3 (d) 0.4
- ii) If two events A and B are independent, then
 (a) $P(A \cap B) = P(A)P(B)$ (b) $P(A + B) = P(A) + P(B)$
 (c) $P(A - B) = P(A)P(B)$ (d) $P(A \cap B) = P(A)P(B/A)$
- iii) A fair die is thrown. The probability that either an odd number or a number greater than 4 will turn up is
 (a) $\frac{2}{5}$ (b) $\frac{3}{7}$ (c) $\frac{2}{7}$ (d) $\frac{2}{3}$
- (iv) The mean of a uniform distribution with parameters a and b is
 (a) $b - a$ (b) $b + a$ (c) $\frac{(a+b)}{2}$ (d) $\frac{b-a}{2}$
- (v) The statistic t is said to be an unbiased estimator of a population parameter θ when
 (a) $E(t) = \theta$ (b) $E(t^2) = \theta$ (c) $E(t^2) = E(\theta)^2$ (d) $E(t)^2 = E(\theta)^2$
- (vi) A random variable X has the following probability density function:

$$f(x) = \begin{cases} kx, & 0 \leq x \leq 1. \\ 0, & \text{otherwise.} \end{cases}$$
 The value of k is
 (a) 1 (b) 2 (c) 4 (d) none of these
- (vii) If a is an element of a group G and order of a is 5, then
 (a) $\circ(a^{10}) = 5$ (b) $\circ(a^{15}) = 5$ (c) $\circ(a^{23}) = 5$ (d) none
- (viii) The order of [6] in the group Z_{14} is
 (a) 2 (b) 14 (c) 7 (d) 5
- (ix) Inverse of the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix}$ is
 (a) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix}$ (d) none
- (x) The number of generators of an infinite cyclic group is
 (a) 1 (b) 2 (c) 0 (d) infinite
- (xi) The generators of the cyclic group $(\mathbb{Z}, +)$ are
 (a) 1, -1 (b) 0, 1 (c) 0, 1 (d) 2, -2
- (xii) Let G be a group and $a \in G$. If $\circ(a) = 17$, then $\circ(a^8)$ is
 (a) 17 (b) 16
 (c) 8 (d) 5
- (xiii) If H is a subgroup of a group G and a, b are two distinct elements of G, then indicate which of the following statements is true
 (a) $aH = Ha$ (b) $Ha \cap Hb = \phi$
 (c) $Ha \cap Hb \neq \phi$ and $Ha \neq Hb$ (d) $aH = bH$

- (xiv) A planar graph has 10 vertices, 6 edges and 3 regions. The number of components of the graph is
 (a) 3 (b) 5 (c) 6 (d) 7
- (xv) If a graph G has chromatic number 8, then G is
 (a) 5-vertex colourable (b) 6- vertex colourable
 (c) 10- vertex colourable (d) none
- (xvi) A complete graph is called Kuratowski's first graph if it has
 (a) 5 vertices (b) 4 vertices (c) 6 vertices (d) 7 vertices
- (xvii) The chromatic number of a graph containing an odd circuit is
 (a) 3 (b) 2 (c) greater than or equal to 3 (d) greater than or equal to 2
- (xviii) Chromatic number of a complete graph with 15 vertices is
 (a) 12 (b) 13 (c) 14 (d) 15
- xix) If t is a statistic such that $E(t^2) = 5$ and $E(t) = 2$, then the standard error of t is
 (a) 0 (b) 1 (c) 2 (d) none of these
- xx) Which one of the following sets forms a group under usual multiplication of complex numbers?
 (a) $\{1, i\}$ (b) $\{1, \omega, \omega^2\}$ (c) $\{1, \omega^2\}$ (d) $\{1, \omega\}$
- xxi) In a Binomial (n, p) distribution, if its mean and variance are 2 and $4/3$ respectively, then the values of n and p are
 (a) 8, $1/4$ (b) 6, $1/3$ (c) 4, $1/2$ (d) none of these
- xxii) If the exponential distribution is given by the probability density function $f(x) = e^{-x}, 0 < x < \infty$, then the mean of the distribution is
 (a) 1 (b) 3 (c) $1/3$ (d) none of these
- xxiii) The order of the dihedral group D_4 is (a) 4 (b) 6 (c) 8 (d) 64
- xxiv) Kuratowski's graph is a (a) planar graph (b) regular graph (c) tree (d) none of these
- xxv) The standard deviation of a sample mean for SRSWR is
 (a) σ^2/n (b) ρ/\sqrt{n} (c) ρ/n (d) n
- xxvi) The standard deviation of a sample mean for SRSWOR is (N is finite)
 (a) σ^2/n (b) ρ/\sqrt{n} (c) ρ/n (d) $\frac{\rho}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}}$

Statistics

- 2) Distribution of marks scored in an examination is normal. Samples of four students' marks are drawn and it is seen that the probability of the sample mean to be less than 61 is 0.44, to be more than 80 is 0.04. Find the mean and S.d of the distribution. [Given that $P(0 < z < a) = 0.06, 0.10, 0.46$ according as $a = 0.15, 0.25$ and 1.75]
- 3) A normal population has a mean 0.1 and standard deviation 2.1. Find the probability that the mean of a sample of size 900 will be negative, Given that $P(|z| < 1.43) = 0.847$
- 4) In measuring the radius of ball bearings manufactured by a machine an engineer estimates that the standard deviation is 0.05 c.m. How large a sample of measurements must be taken in order to be 95% confident that the error of his measurement of average radius will not exceed 0.01 c.m
- [Given $\int_0^{1.96} \phi = 0.475$],
- 5) The marks obtained by 17 students in an examination have a mean 57 and variance 64. Find 99% confidence interval for the mean of the population of marks assuming it to be normal.
 [Given that $P(t > 3.250) = 0.005$ for 16 d.fs]

- 6) From a normal distribution of variance 5 samples are drawn of size 20. Find the mean and standard deviation of the sampling distribution of the sample variance. Find the probability that the sample variance lies between 8.21 and 9.645. [Given $\chi^2_{0.025;19} = 32.84, \chi^2_{0.005;19} = 38.58$. [Ans. 19/4, 19/8, 0.0215]
- 7) If T is an unbiased estimate of θ , then show that \sqrt{T} is a biased estimate of $\sqrt{\theta}$.
- 8) If T_1 and T_2 be statistic with expectation $E(T_1) = 2\theta_1 + 3\theta_2$ and $E(T_2) = \theta_1 + \theta_2$, find unbiased estimators of the parameters θ_1 and θ_2 .
- 9) If $x_1, x_2, x_3, x_4, x_5, x_6$ be an independent simple random sample from a normal population with unknown variance σ^2 , find K so that $K[(X_1 - X_2)^2 + (X_3 - X_4)^2 + (X_5 - X_6)^2]$ is an unbiased estimator of σ^2 .
- 10) If a population has Poisson distribution with parameter λ , then show that the sample mean is the maximum likelihood estimate of λ .
- 11) If a population has normal distribution with parameter μ and σ , then show that the sample mean \bar{x} is maximum likelihood estimate of μ , where σ is known. Hence prove that this estimate is unbiased and consistent.
- 12) Find the maximum likelihood estimates for population having Binomial distribution.
- 13) If a population has normal distribution with parameter μ and σ , then prove that the statistic $\frac{1}{n} \sum_{i=1}^n (x_i - \mu)^2$ is maximum likelihood estimate of σ^2 where μ is known.
- 14) The weight in pound of a sample of 12 packets of butter are 13, 7, 22, 15, 12, 18, 14, 21, 8, 17, 10, 23 taken at random from its population having standard deviation 5. Find 95% confidence interval for the mean of the population. [Given $Z_{0.05} = 1.96$]
- 15) If S^2 be the sample variance of a sample of size n drawn from a population with mean μ and standard deviation ρ , then show that $E(S^2) = \frac{n-1}{n} \rho^2$ in SRSWR..
- 16) A random sample with observations 65, 71, 64, 71, 70, 69, 64, 63, 67, 68 is drawn from a normal population with standard deviation $\sqrt{7.056}$. Test the hypothesis that the population mean is 69 at 1% level of significance. [Given that $P(0 < z < 2.58) = 0.495$]
- 17) A die is thrown 150 times with the following result:
- | | | | | | | |
|-----------------|----|----|----|----|----|----|
| No. turned up : | 1 | 2 | 3 | 4 | 5 | 6 |
| Frequency : | 19 | 23 | 28 | 17 | 32 | 31 |
- Test the hypothesis that the die is unbiased. Given $\chi^2_{0.05;5} = 11.07$ and $\chi^2_{0.05;6} = 12.59$.